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NUMERICAL INVESTIGATION OF THE TEMPERATURE FIELD OF A DAM WITH FREEZING COLUMNS

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The problem of interconnected heat and mass transfer is numerically solved. The temperature field of a dam with an artificial freezing antifiltration curtain is determined.

In the construction of dams in the permafrost region the problem of the thermal stability of the soils arises because when they melt, they lose their load-bearing capacity and become highly water-permeable. To maintain the soils in the foundation and in the body of the dam in their frozen state, and thereby also to prevent losses on filtering, soils artificially frozen with the aid of number of boreholes are at present widely used: in some cases by stimulating circulation of natural cold, in others with the aid of refrigeration techniques.

Several authors in the USSR and other countries studied in recent times the processes of freezing of the soil, with filtering of the moisture taken into account. For instance, Melamed and Medvedev [1] studied the process of freezing of finely dispersed soil taking bulging into account on condition that the phase boundary between water and ice the moisture is constant and equal to its lower limit. Frivik and Komini [2] presented the results of experimental and numerical investigation of the process of freezing of the soil with a view to the filtration flows of moisture (the equation of heat conduction with a convective term is solved, where the speeds are determined from Darcy's law) with constant moisture equal to saturation moisture; on the surface of the refrigeration plant, boundary conditions of the first kind are specified.

In the present article we examine the problem of determining the temperature field of a dam with an artificially provided antifiltration curtain. Mass transfer in the melting zone considerably complicates the investigation because it makes it necessary to take into account the interaction of the temperature and moisture fields in the melting and frozen zones of

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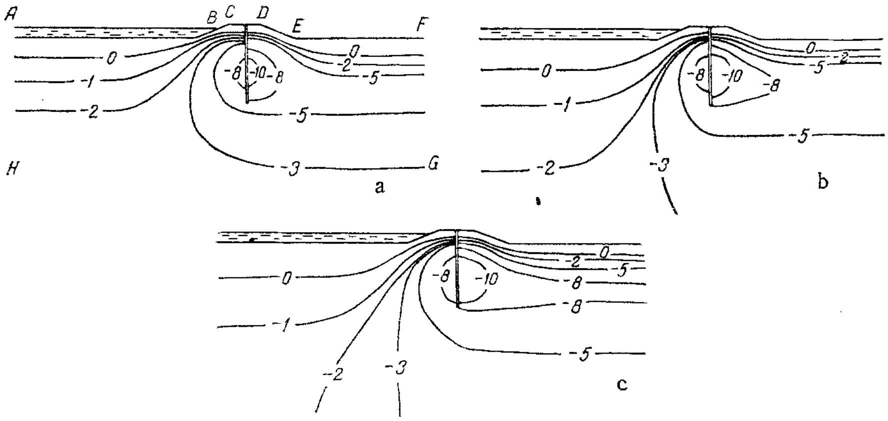

Fig. 1. Temperature field of a dam without insulation with constant thermal flux from a refrigerating plant $Q_1 = -1.163$ $W/(m^2 \cdot {}^\circ K)$: a) after 9 months; b) after 1 year and 9 months; c) after 2 years and 9 months; $T_a = 16°C$.

the soil. Mathematically this problem reduces to the solution of the following system of equations for the functions u_i (temperature) and w_i (moisture content):

$$c_1(u_1)\rho(u_1)\frac{\partial u_1}{\partial t} = \operatorname{div}(\lambda_1(u_1)\operatorname{grad} u_1), \quad (x_1, x_2) \in \Omega_1, \quad t > 0, \tag{1}$$

$$c_i(u_i, w_i)\rho(u_i, w_i)\frac{\partial u_i}{\partial t} = \operatorname{div}(\lambda_i(u_i, w_i)\operatorname{grad} u_i + \delta_w \eta(u_i - u^*) k_i(w_i)\operatorname{grad} w_i), \tag{2}$$

$$\frac{\partial w_i}{\partial t} = \operatorname{div}(k_i(w_i)\eta(u_i - u^*)(\operatorname{grad} w_i + \delta_T \operatorname{grad} u_i)),$$
$$(x_1, x_2) \in \Omega_i, \quad t > 0, \quad i = 2, 3, \tag{3}$$

with the initial conditions

$$u_i(x_1, x_2, 0) = u_0 = \text{const}, \quad w_i(x_1, x_2, 0) = w_0 = \text{const} \tag{4}$$

and the boundary conditions

$$-\lambda_i(u_i, w_i)\frac{\partial u_i}{\partial n} = F(u_i), \ (x_1, x_2) \in \partial\Omega_4, \ t > 0, \ i = 2; 3, \tag{5}$$

depending on the kind of refrigeration plant, $F(u_i)$ is equal to Q_1 or $\alpha_{ef}(T_B(t) - u_i)$;

$$\lambda_i \frac{\partial u_i}{\partial n} = \lambda_{i+1}\frac{\partial u_{i+1}}{\partial n}, \ u_i(x_1, x_2, t) = u_{i+1}(x_1, x_2, t), \ i = 1; 2,$$
$$w_{i+1}(x_1, x_2, t) = w_s', \ i = 2, \ (x_1, x_2) \in (\partial\Omega_i \cap \partial\Omega_{i+1}), \ t > 0,$$
$$w_2(x_1, x_2, t) = w_3(x_1, x_2, t), \tag{6}$$

$$k_2(w_2)\frac{\partial w_2}{\partial n} = k_3(w_3)\frac{\partial w_3}{\partial n}, \ (x_1, x_2) \in (\partial\Omega_2 \cap \partial\Omega_3), \ t > 0, \tag{7}$$

$$u_3(x_1, x_2, t) = 4, \quad w_3(x_1, x_2, t) = w_s, \quad (x_1, x_2) \in (\partial\Omega_1 \cap \partial\Omega_3), \tag{8}$$

$$-\lambda_i \frac{\partial u_i}{\partial n} = \alpha(T_a(t) - u_i), \ i = \overline{1, 3}, \quad w_i(x_1, x_2, t) = w_1, \tag{9}$$
$$i = 2; 3, \quad (x_1, x_2) \in \Gamma_1, \quad t > 0,$$
$$-\lambda_i \frac{\partial u_i}{\partial n} = 0, \quad i = 1; 3,$$
$$-k_3(w_3)\frac{\partial w_3}{\partial n} = 0, \quad (x_1, x_2) \in \Gamma_2, \quad t > 0. \tag{10}$$

On the movable boundary of phase transition Stefan's condition is fulfilled

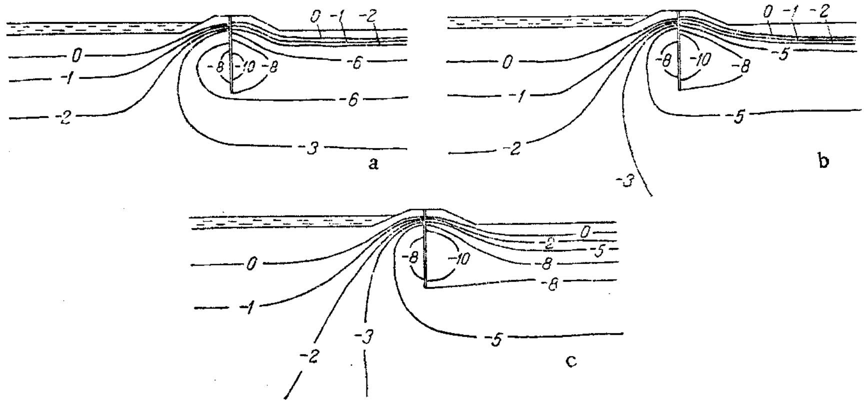

Fig. 2. Temperature field of a dam with partial insulation over CDE with constant thermal flux Q_1 from a refrigerating plant (a, b, c: see Fig. 1).

$$\left[\lambda_1(u_1)\frac{\partial u_1}{\partial n}\right]_{\xi(t)} = \gamma\rho_1\frac{d\xi}{dt},$$

$$u_{1m}(x_1,\ x_2,\ t) = u_{1M}(x_1,\ x_2,\ t) = u^*,\quad (x_1,\ x_2)\in\xi(t), \tag{11}$$

$$\left[\lambda_i(u_i,\ w_i)\frac{\partial u_i}{\partial n}\right]_{\xi(t)} = \gamma\rho_i w_i\frac{d\xi}{dt} + \gamma\rho_i k_i(w_i)\frac{\partial w_i}{\partial n},$$

$$u_{im}(x_1,\ x_2,\ t) = u_{iM}(x_1,\ x_2,\ t) = u^*,\ i=2;\ 3,\ (x_1,\ x_2)\in\xi(t), \tag{12}$$

where

$$\eta(x) = \begin{cases} 1, & x>0, \\ 0, & x\leqslant 0. \end{cases}$$

For the thermophysical characteristics the following dependences were adopted, data of [3-5] being used:

$$k(w) = 3{,}786\cdot 10^{-5}\exp(16{,}446\,w),$$

$$\rho(u,\ w) = \rho_{sk}(1+w),\quad c(u,\ w) = (c_{sk}+c_1(u)\,w)/(1+w),$$

$$\lambda_{im}(u_i,\ w_i) = \lambda_{sk} + bw_i,\quad \lambda_{iM}(u_i,\ w_i) = \lambda_{im}(u_i,\ w_i)/(0{,}67+4{,}1\,w_i).$$

With the aid of the enthalpy function and the through-counting method [6, 7], the problem reduces to the solution of a system of equations with smoothed coefficients:

$$\tilde{c}_1(u_1)\,\tilde{\rho}_1(u_1)\frac{\partial u_1}{\partial t} = \operatorname{div}(\tilde{\lambda}_1(u_1)\operatorname{grad}u_1),\quad (x_1,\ x_2)\in\Omega_1,$$

$$\tilde{c}_i(u_i,\ w_i)\,\tilde{\rho}_i(u_i,\ w_i)\frac{\partial w_i}{\partial t} = \operatorname{div}(\tilde{\lambda}_i(u_i,\ w_i)\operatorname{grad}u_i + \tilde{k}_{wi}(w_i)\operatorname{grad}w_i),$$

$$\frac{\partial w_i}{\partial t} = \operatorname{div}(\tilde{k}_i(w_i)(\operatorname{grad}w_i + \delta_T\operatorname{grad}u_i)),$$

$$(x_1,\ x_2)\in\Omega_i,\quad i=2;\ 3,\quad t>0,$$

where

$$\tilde{\lambda}_i = \begin{cases} \lambda_{iM}(u_i,\ w_i), & u_i<u^*-\Delta, \\ (\lambda_{im}+\lambda_{iM}+(\lambda_{im}-\lambda_{iM})(u_i-u^*)/\Delta)/2, & u^*-\Delta\leqslant u_i\leqslant u^*+\Delta, \\ \lambda_{im}(u_i,\ w_i), & u_i>u^*+\Delta, \end{cases}$$

$$\tilde{c}_i\tilde{\rho}_i = \begin{cases} c_{iM}(u_i,\ w_i)\,\rho_{iM}(u_i,\ w_i), & u_i<u^*-\Delta, \\ (c_{iM}\rho_{iM}+c_{im}\rho_{im}+(c_{im}\rho_{im}-c_{iM}\rho_{iM})(u_i-u^*)/\Delta)/2 + \\ \quad +\gamma\rho_i w_i(1-|u_i-u^*|/\Delta)/\Delta, & u^*-\Delta\leqslant u_i\leqslant u^*+\Delta, \\ c_{im}(u_i,\ w_i)\,\rho_{im}(u_i,\ w_i), & u_i>u^*+\Delta, \end{cases}$$

$$\tilde{k}_i(w_i)=\begin{cases} k_i(w_i), & u_i>u^*+\Delta, \\ k_i(w_i)(u_i-u^*+\Delta)/2\Delta, & u^*+\Delta\geqslant u_i\geqslant u^*-\Delta \\ 0, & u_i<u^*-\Delta, \end{cases}$$

$$\tilde{k}_{wi}(w_i)=\begin{cases} 0, & u_i<u^*-\Delta, \\ k_i(w_i)(u_i-u^*+\Delta)/\Delta, & u^*-\Delta\leqslant u_i\leqslant u^*, \\ k_i(w_i)(1-(1-\delta_w)(u_i-u^*)/\Delta), & u^*<u_i\leqslant u^*+\Delta, \\ \delta_w k_i(w_i), & u_i>u^*+\Delta, \end{cases}$$

with the corresponding initial conditions (4) and the boundary conditions (5)-(10); it is solved numerically by the finite difference method using locally one-dimensional schemes and the method of simple iterations:

$$\tilde{c}_1^{n+1/2,s}\tilde{\rho}_1^{n+1/2,s}u_{1\bar{t}}^{n+1/2,s+1}=(\tilde{\lambda}_1^{n+1/2,s}u_{1\bar{x}_1}^{n+1/2,s+1})_{\hat{x}_1},\quad (x_1,\ x_2)\in\Omega_1,$$

$$\tilde{c}_i^{n+1/2,s}\tilde{\rho}_i^{n+1/2,s}u_{i\bar{t}}^{n+1/2,s+1}=(\tilde{\lambda}_i^{n+1/2,s}u_{i\bar{x}_1}^{n+1/2,s+1}+\tilde{k}_{wi}^{n+1/2,s}w_{i\bar{x}_1}^{n+1/2,s})_{\hat{x}_1},$$

$$w_{i\bar{t}}^{n+1/2,s+1}=(\tilde{k}_i^{n+1/2,s}(w_{i\bar{x}_1}^{n+1/2,s+1}+\delta_T u_{i\bar{x}_1}^{n+1/2,s+1}))_{\hat{x}_1},$$

$$(x_1,\ x_2)\in\Omega_i,\quad i=2;\ 3,$$

$$\tilde{c}_1^{n+1,s}\tilde{\rho}_1^{n+1,s}u_{1\bar{t}}^{n+1,s+1}=(\tilde{\lambda}_1^{n+1,s}u_{1\bar{x}_2}^{n+1,s+1})_{\hat{x}_2},\quad (x_1,\ x_2)\in\Omega_1,$$

$$\tilde{c}_i^{n+1,s}\tilde{\rho}_i^{n+1,s}u_{i\bar{t}}^{n+1,s+1}=(\tilde{\lambda}_i^{n+1,s}u_{i\bar{x}_2}^{n+1,s+1}+\tilde{k}_{wi}^{n+1,s}w_{i\bar{x}_2}^{n+1,s})_{\hat{x}_2},$$

$$w_{i\bar{t}}^{n+1,s+1}=(\tilde{k}_i^{n+1,s}(w_{i\bar{x}_2}^{n+1,s+1}+\delta_T u_{i\bar{x}_2}^{n+1,s+1}))_{\hat{x}_2},\quad (x_1,\ x_2)\in\Omega_i,\ \ i=2;\ 3,$$

$$\tilde{\lambda}_i^{n+k/2,s}u_{ix_k}^{n+k/2,s+1}=0,\quad i=1;\ 3,$$

$$\tilde{k}_3^{n+k/2,s}w_{3x_k}^{n+k/2,s+1}=0,\quad (x_1,\ x_2)\in\Gamma_2,$$

$$\tilde{\lambda}_i^{n+k/2,s}u_{i\bar{x}_k}^{n+k/2,s+1}=\cos(\mathbf{n},\hat{\ }\mathbf{x}_k)\,\alpha\,(u_i^{n+k/2,s+1}-T_a),\ \ i=\overline{1,\ 3},$$

$$w_i^{n+k/2,s+1}=w_1,\quad i=2,\ 3,\quad (x_1,\ x_2)\in\Gamma_1,$$

$$\tilde{\lambda}_i^{n+k/2,s}u_{i\bar{x}_k}^{n+k/2,s+1}=F(u_i^{n+k/2,s+1}),\quad (x_1,\ x_2)\in\partial\Omega_4,$$

$$\tilde{\lambda}_i^{n+k/2,s}u_{i\bar{x}_k}^{n+k/2,s+1}=\tilde{\lambda}_{i+1}^{n+k/2,s}u_{i+1x_k}^{n+k/2,s+1},$$

$$u_i^{n+k/2,s+1}=u_{i+1}^{n+k/2,s+1},\ \ i=1;\ 2,\ \ (x_1,\ x_2)\in(\partial\Omega_1\cap\partial\Omega_2)\cup(\partial\Omega_2\cap\partial\Omega_3),$$

$$w_i^{n+k/2,s+1}=w_s,\ \ (x_1,\ x_2)\in(\partial\Omega_i\cap\partial\Omega_1),\ \ i=2;\ 3,\ \ k=1;\ 2.$$

We carried out numerical calculations for the case that the body of the dam and the foundations consist of loam and sandy loam; we had for sandy loam: ρ_{sk} = 1600, c_{sk} = 0.733, λ_{sk} = 1.29, b = 3.84; for loam: ρ_{sk} = 1500, c_{sk} = 0.775, λ_{sk} = 0.72, b = 3.72; u_0 = —2.5°C, w_0 = 0.1, w_a = 0.2, w_1 = 0.05, γ = 333.7, α = 18.34, δ_T = δ_W = 0, $T_a(t)$ = —7.4 + 24.1 cos $(2\pi(t + t_0)/t_1)$.

The results of the calculations of long-term isotherms are presented in Figs. 1-3.

Figure 1 shows the temperature fields of a dam for different periods of time, with constant thermal flux from a refrigerating plant F(u) = Q_1 = —1.163 W/m^2 in the absence of insulation on the surface CDE. The large heat capacity of the reservoir maintains thawed ground under its bottom to a depth of about 2.5-3 m. It can be seen from Fig. 2 which shows the isotherms for the same periods of time and with the same thermal flux, but with partial heat insulation of the surface of the dam CDE with thermal resistance R = 0.298 and heat-transfer coefficient with the air α = 2.84 W/(m$^2\cdot$°K), that the depth of thawing is smaller than in Fig. 1 solely at the time of the first summer period. Later the effect of the heat

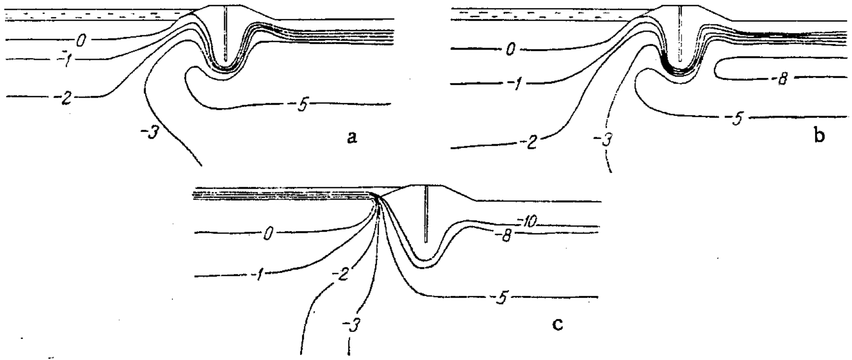

Fig. 3. Temperature field of a dam with refrigerating plant for liquid, with α_{ef} = 15.77 W/(m·°K): a) after 9 months; b) after 1 year and 9 months, T_a = 16°C; c) after 2 years and 3 months, T_a = —31°C.

insulation on the temperature field of the dam and of its foundation becomes insignificant because the depth of freezing in the subsequent winter periods becomes smaller.

Upon freezing of the dam with a siphon pump we calculated from the conditions of thermal balance the effective heat-transfer coefficient which for circulation of kerosene with a speed of v = 0.01 m/sec is equal to 15.77 W/(m²·°K). The results of the calculations, where on the surface of the freezing column a boundary condition of the 3rd kind with α_{ef} was specified, are shown in Fig. 3. In winter a thick frozen core forms, the thawed zone is maintained under the water reservoir only, but in summer the convective heat exchange of the siphon pump leads to the thawing of the body of the dam if adequate measures are not taken.

NOTATION

t, time; x_1, x_2, space coordinates; Ω_1, water reservoir; Ω_2, body of the dam; Ω_3, foundation of the dam; Ω_4, freezing column; u_1, temperature of the water reservoir; u_2, w_2, temperature and moisture content, respectively, of the body of the dam; u_3, w_3, temperature and moisture content, respectively, of the dam foundation; c_i, ρ_i, λ_i, coefficient of heat capacity, density, thermal conductivity, respectively, of water (i = 1), of the materials of the body (i = 2), and of the foundation (i = 3) of the dam; $k_i(w_i)$, moisture conductivity; u_0, w_0, initial temperature and moisture, respectively; w_s, moisture content in case of saturation; α, heat-transfer coefficient with air; $T_a(t)$, air temperature; γ, heat of phase transition of water; u^*, temperature of phase transition; Δ, smoothing parameter; ρ_{sk}, c_{sk}, λ_{sk}, density, heat capacity, and thermal conductivity of the soil skeleton, respectively; Γ_1, boundaries of the dam, of the surface of the water reservoir, and of the dam foundation in contact with the atmosphere (along the line ABCDE); Γ_2, part of the boundaries of the water reservoir and of the dam foundation along the line AHGF.

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